Appendix D

DIAPHRAGMATIC BEHAVIOUR
OF MULTISTOREY SPACE FRAMES
GENERAL CASE

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D.1 Subject description

The assessment of the behaviour of one-storey plane frames under horizontal seismic forces was presented in paragraph 5.1, where the structural unit examined was the column (or wall). In appendix B', the crossbar was used to assess the behaviour of the multistorey plane frame.

In this chapter, the composition of space frames, through beams and slabs, is considered. The diaphragm is the structural unit for the assessment of coupled space frames.

Studying paragraph 5.4 and Appendices B and C is recommended in order to better understand this chapter.

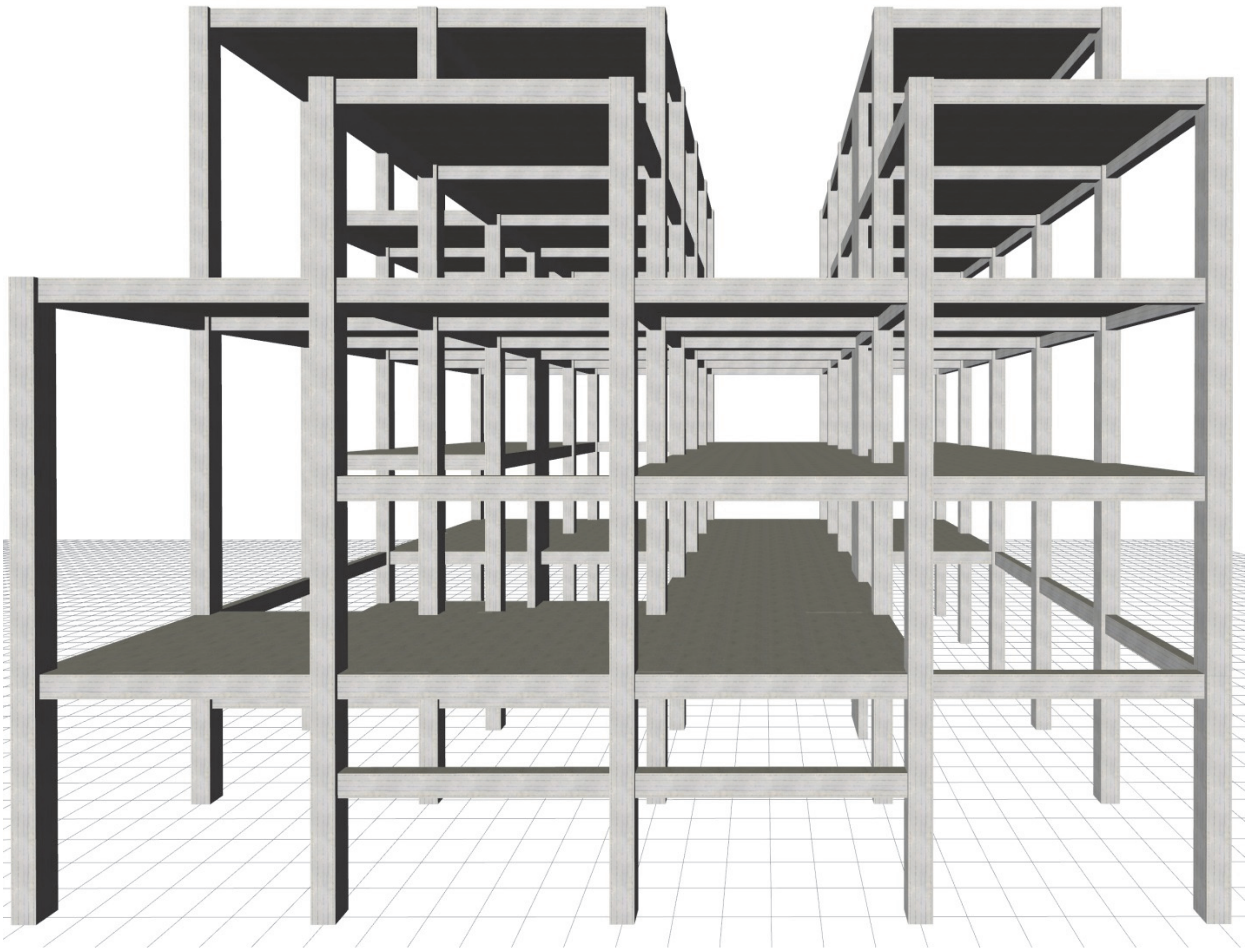

Figure D.1-1: The structural frame with 6 levels, project <B_d1>

<u>A diaphragm</u> is the horizontal part of the floor consisting of slabs, connecting beams and columns. A floor may comprise more than one diaphragm, as shown in the example building of the present paragraph.

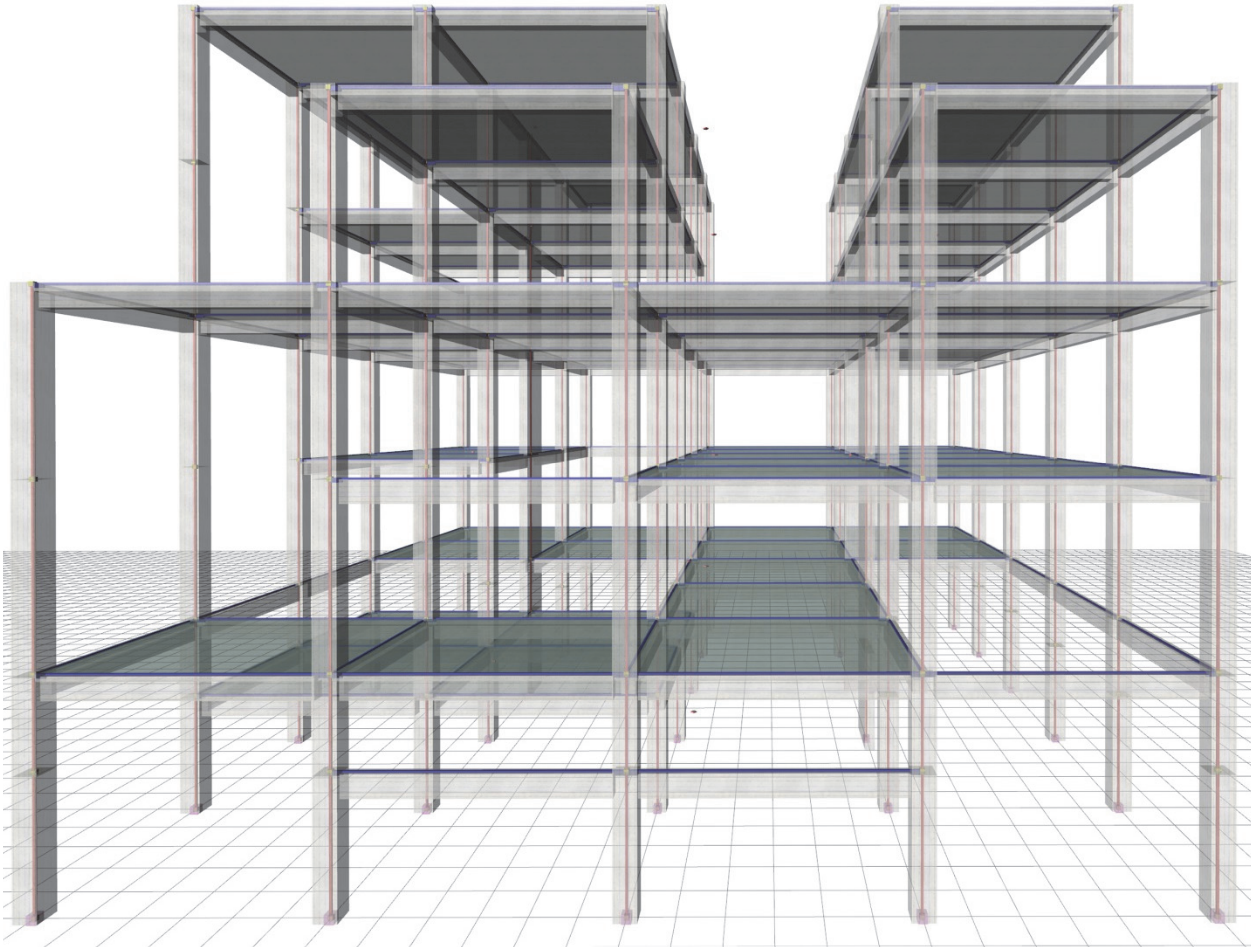

Figure D.1-2: The space frame structural model with bars of columns and beams and diaphragms

The most characteristic point of the diaphragm is the centre of stiffness, described in paragraphs 3.1.4, 5.4.3.4 and C.5.

The <u>centre of stiffness</u> C_T of a specific floor diaphragm is defined as the point about which the diaphragm rotates, under horizontal seismic force H.

The C_T point depends only on the geometry of the floor and is independent of the loadings, meaning that regardless the magnitude of the force, which may become equal to 2H, or 3H, or any other value, the centre of stiffness will remain the same. Of course, it depends on the geometry of the overlying and the underlying floors.

<u>Principal axis system of diaphragm</u> is defined as the orthogonal axis system xC_Ty, in which when a horizontal seismic force H is applied on C_T, along axis x (or y), it results in translating C_T only along axis x (or y respectively). The angle a of the principal system, as to the initial system X0Y, is called principal angle of the diaphragm.

The above diaphragm data as well as the torsional stiffness K_θ and torsional ellipse stiffness ellipse (C_T, r_x, r_y), should be calculated in a general way. The method to be used should deal with more than one diaphragm per floor. Moreover, the general solution should account for the influence of non-diaphragmatic frames as well as the uneven level foundation. All possible special cases met in practice are included in the example presented in this paragraph, except for foundations, just for simplification reasons. Full scale analysis (including the foundation), can be performed by the engineer using the related software.

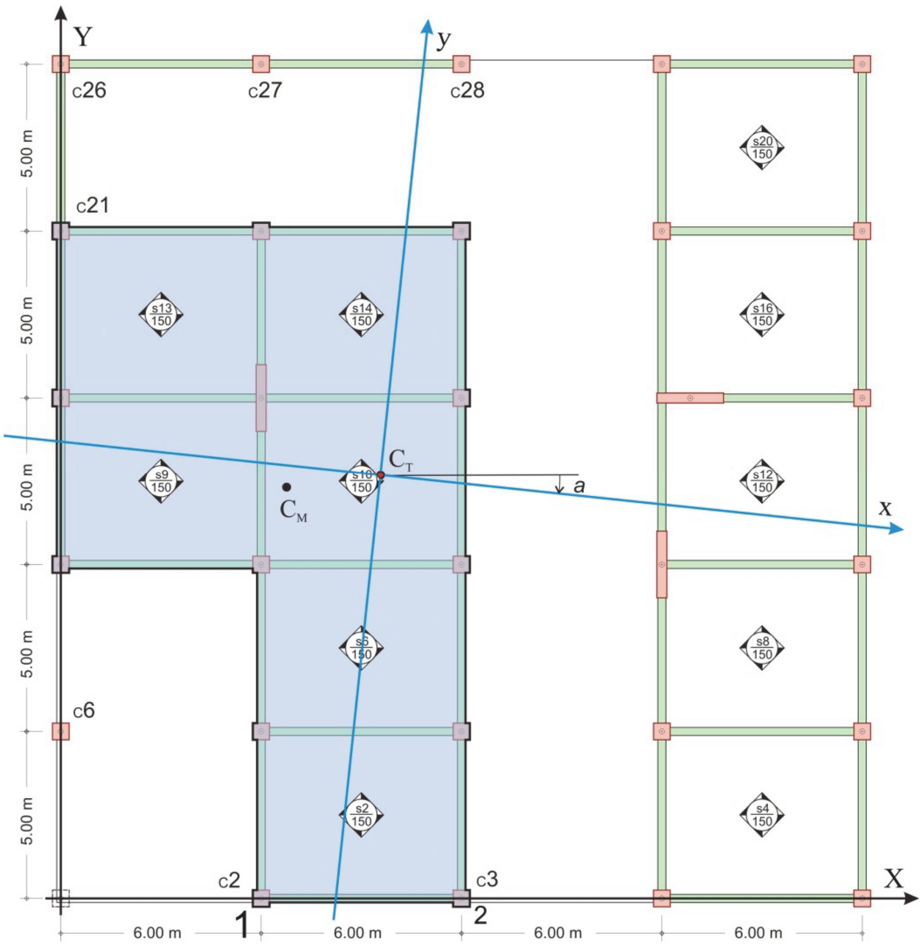

Figure D.1-3: Floor plan of 5th level and diaphragm on the left examined next.
The cross-sections of columns, walls and beams are 500/500, 2000/300 and 300/500 respectively.

The behaviour of diaphragm on the left at the 5th level, analysed next is directly affected by the L-type frame C_{21}-C_{26}-C_{27}-C_{28} connected to it and indirectly by column C_6 and the diaphragm on the right, through the frames of other floors.

A general method for the calculation of the diaphragm data is presented next.

This method creates a condition allowing the diaphragm only to rotate about the centre of stiffness C_T, which remains stationary with respect to the ground.

The method comprises four steps, the first three of which include a space frame analysis.

D.2 General method for the calculation of the diaphragm i data

Step 1: Analysis under force H_X and concentrated moment $M_{CM,X}$ applied at the centre of mass of diaphragm i.

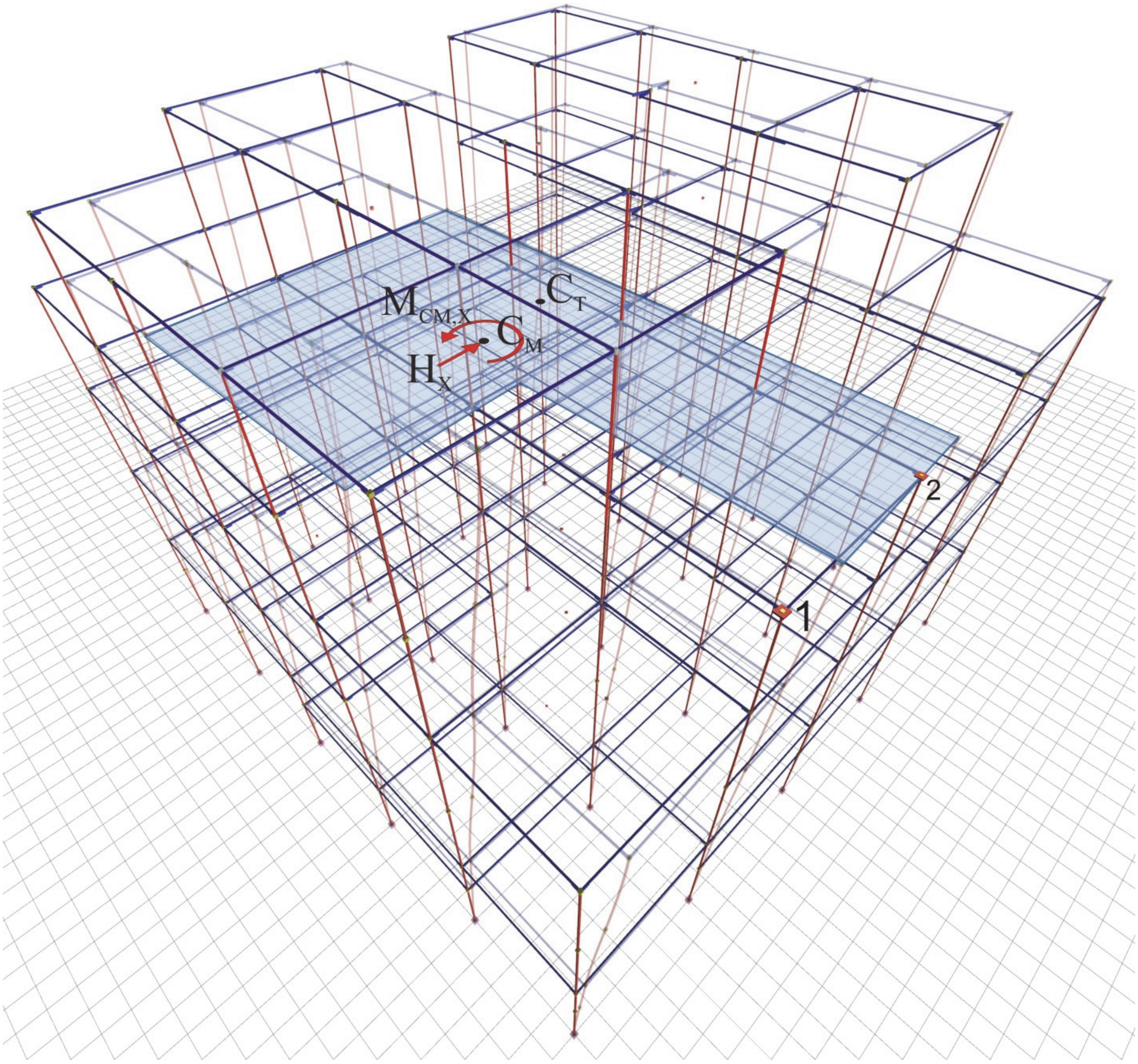

Figure D.2-1: Loading: Force H_X=100 and moment $M_{CM,X}$ on centre of mass C_M of diaphragm i
Results: total displacements of structure and displacements of diaphragm i

The two displacements δ_{XX1}, δ_{XY1} of point 1 of diaphragm i and its angle θ_{XZ} are required.
Any value may be given to force H_X, as long as it is the same for the analyses of the three first steps. In this example, H_X is given equal to $100\ kN$. The application point of force H is irrelevant, but for higher accuracy in calculations, the force is assumed applied at an arbitrary eccentricity c_y, e.g. $2.0\ m$, in relation to the centre of mass, in order to produce a significant rotation of the diaphragm and thus achieve a higher accuracy in the calculations. That eccentricity is essential, especially in cases where the centre of stiffness is close or coincides with the centre of mass. Therefore, in addition to force H_X, moment $M_{CM,x}=100kN\cdot 2.00m= 200\ kNm$ is also applied at C_M.

The magnitude of force H needs to be important enough to result in significant diaphragm translations that can be distinguished from the translations due to moment.
The application of the sole moment in the absence of force H results in rotation, but also in translations, which cannot be calculated since the centre of stiffness is still unknown.
Point 1 corresponds to column K2 joint, but could be any point of the diaphragm.

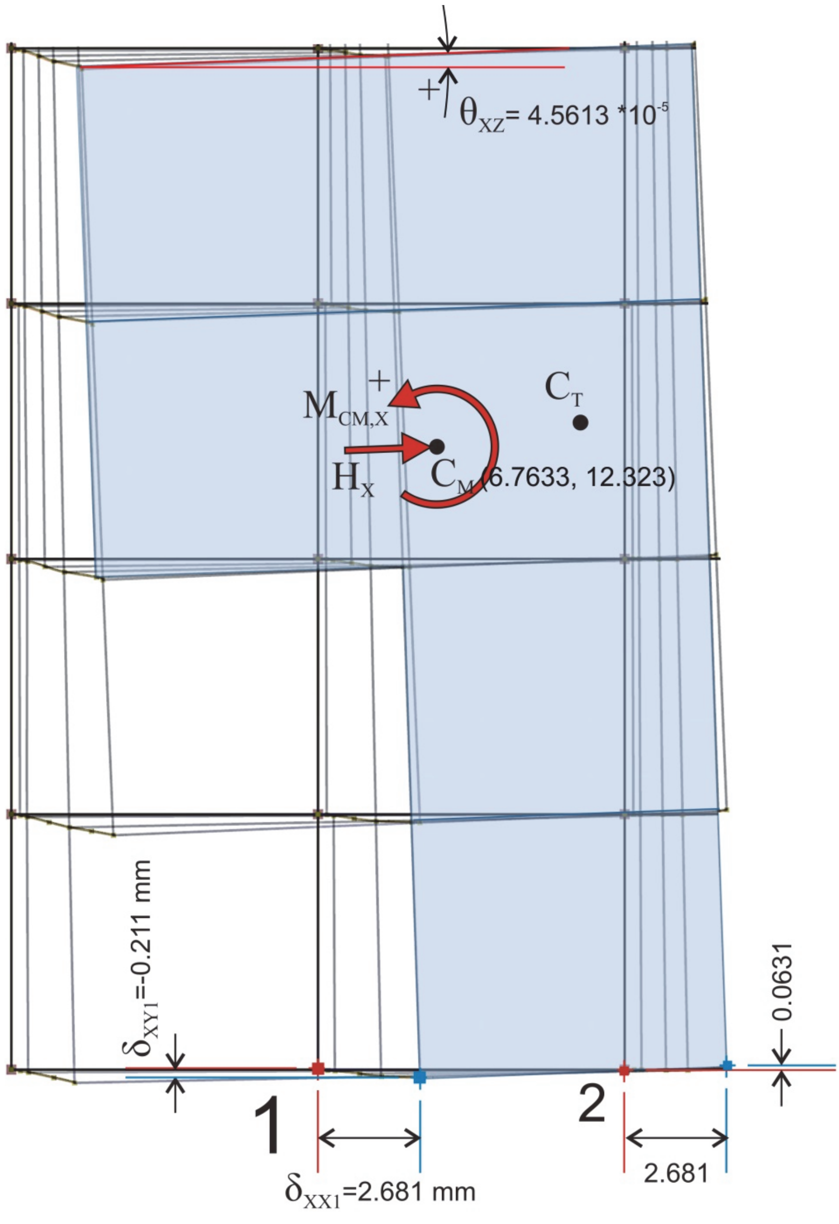

Figure D.2-2: The diaphragm displacements in plan, due to 2 parallel translations δ_{XXo}, δ_{XYo} and one rotation θ_{XZ}

The only results needed from this analysis are the translations of point 1 $\delta_{XX1}=2.681\ mm$, $\delta_{XY1}=-0.231\ mm$ and the angle $\theta_{XZ}=4.5613\times10^{-5}$ of the diaphragm. The displacements of point 2 will be used only to verify the generality of the method.

All displacements are absolute with respect to the ground.

Step 2: Analysis under force H_X applied at the centre of mass of diaphragm i with rotational restraint.

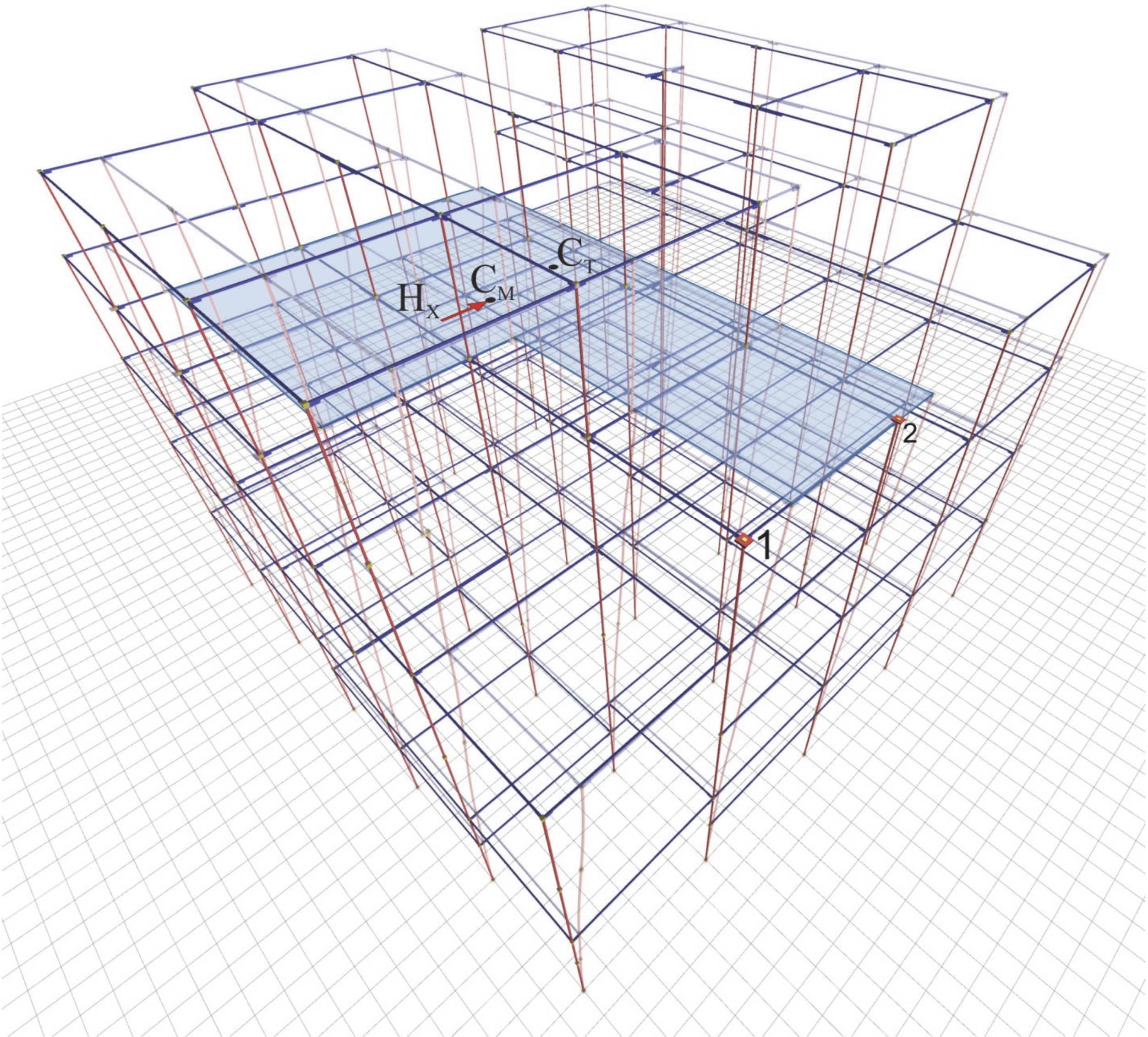

Figure D.2-3: Loading: H_X=100 on diaphragm i with rotational restraint

Results: the total displacements of structure and the two parallel translations of diaphragm

The two translations of point 1 δ_{XXo}, δ_{XYo}, are only required, being identical for any point of the diaphragm i, thus also for C_T, since the angle of rotation of the diaphragm is zero.

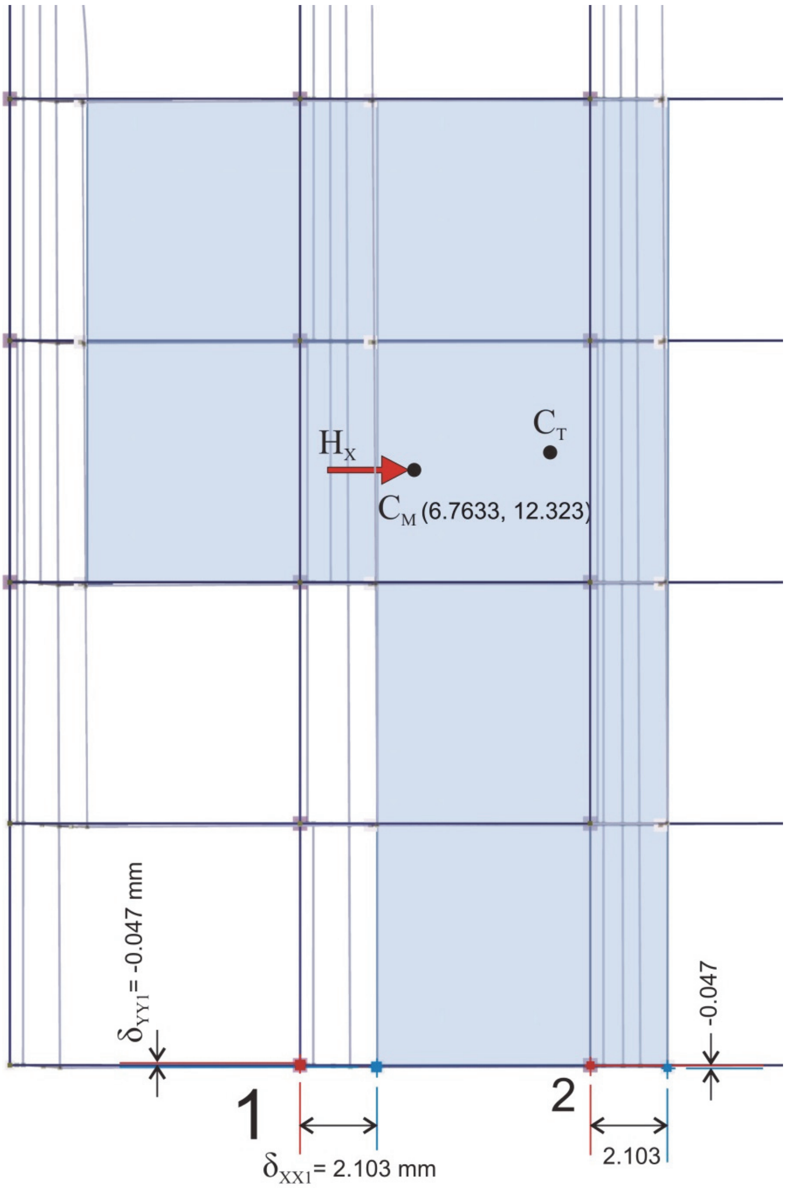

Figure D.2-4: The 2 parallel translations of the diaphragm δ_{XXo}, δ_{XYo}, in plan.

All diaphragm points, thus also CT, have identical displacements.

Diaphragm is restrained against rotation, therefore θ_{XZ}=0.

Due to zero rotation of diaphragm i, point 1 has only two parallel translations, $\delta_{XXo}=2.103\ mm$ and $\delta_{XYo}=-0.047mm$, being identical for all diaphragm points, thus also for C_T. In other diaphragms located in the same, upper or lower level, small, yet measurable rotations exist, thus every point on them has different displacements.

Step 3: Analysis under force H_Y applied at the centre of mass of diaphragm i with rotational restraint

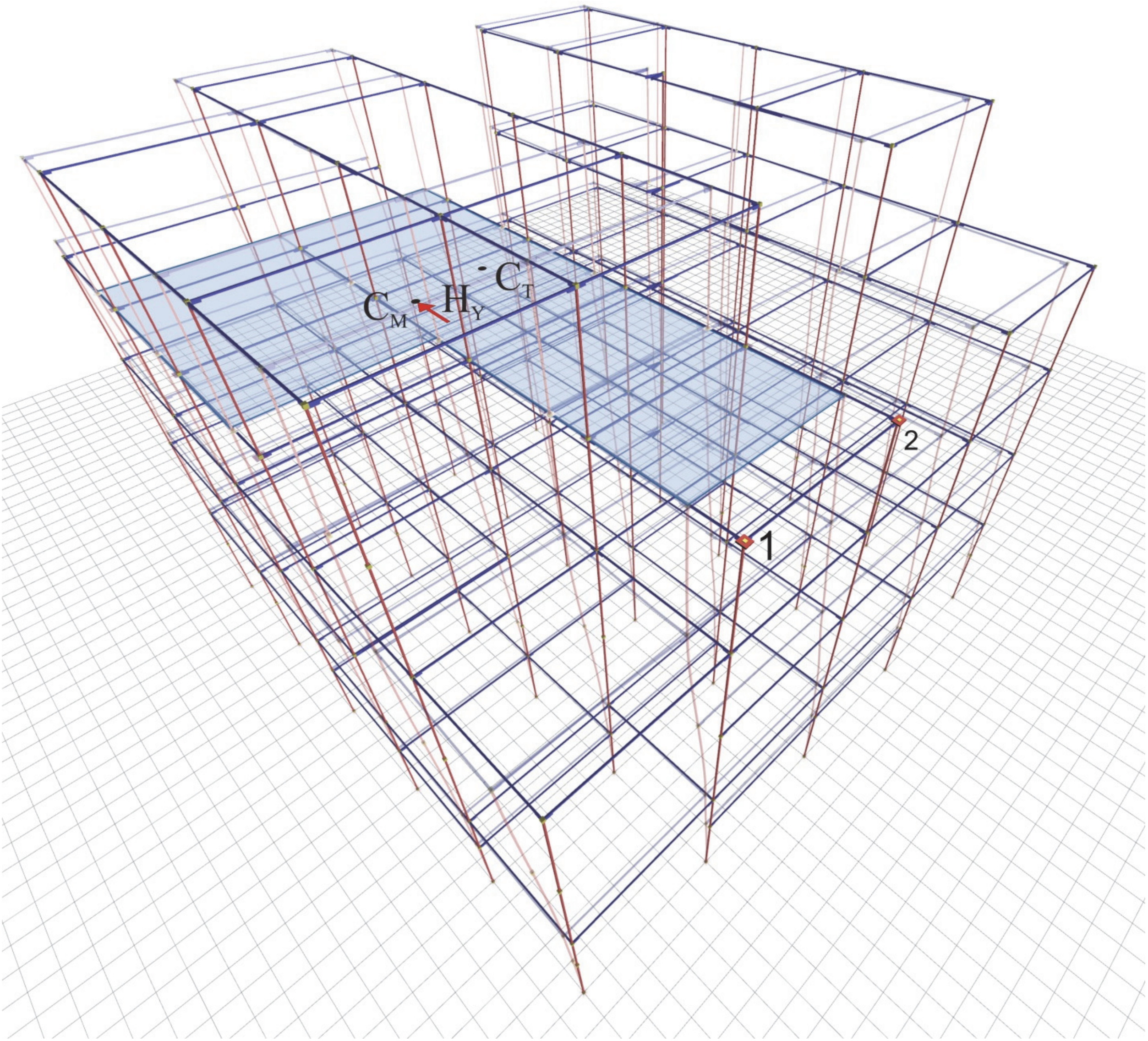

Figure D.2-5: Loading: H_Y=100 on diaphragm i with rotational restraint Results: the total displacements of structure and the two parallel translations of diaphragm

The two translations of point 1 δ_{XXo}, δ_{XYo}, are only required, being identical for any point of the diaphragm i, thus also for C_T, provided that the angle of rotation of the diaphragm is zero.

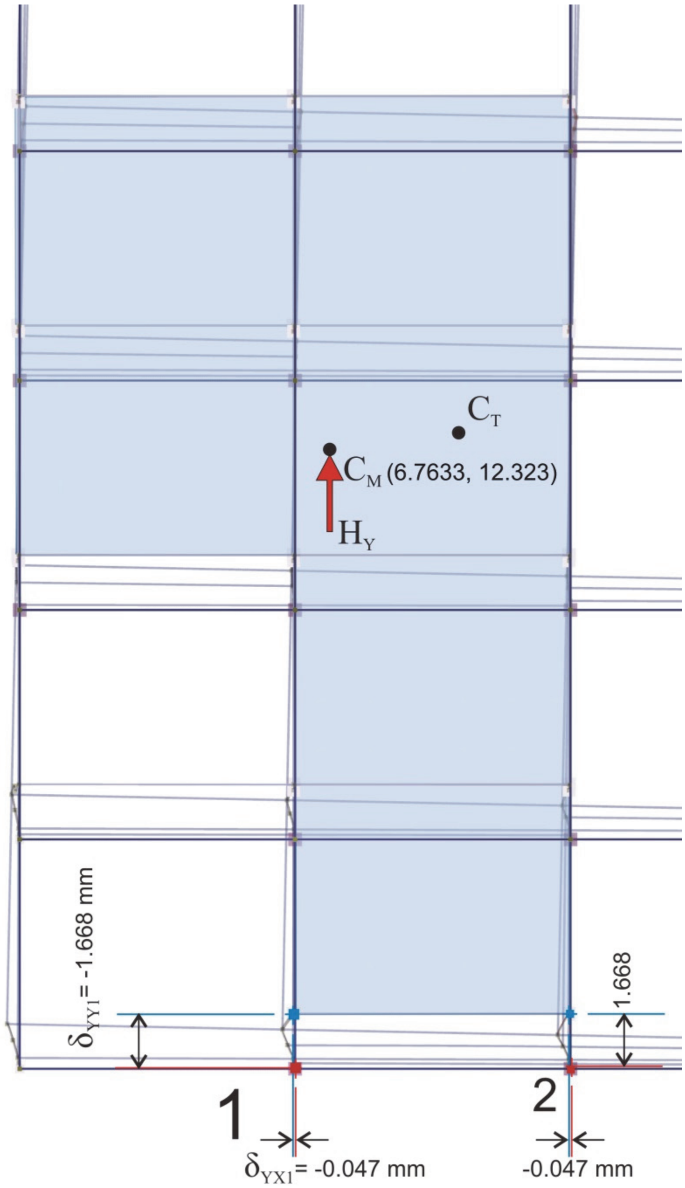

Figure D.2-6: The 2 parallel diaphragm translations δ_{XXo}, δ_{XYo}, in plan.

All diaphragm points, thus also C_T, have identical displacements.

Diaphragm is restrained against rotation, therefore θ_{XZ}=0.

Due to zero rotation of diaphragm i, point 1 has only two parallel translations, $\delta_{YXo}=-0.047$ and $\delta_{YYo}=1.668$ *mm*, being identical for all diaphragm points, thus also for C_T. As in step 2, in other diaphragms located in the same, upper or lower level small, yet measurable rotations exist, thus every point on them has different displacements.

The secondary displacements δ_{XYo} of loading 1 and δ_{YXo} of loading 3 are always equal, i.e. $\delta_{XYo}=\delta_{YXo}$ should always derive from the space frame analysis.

Analysis 3 is performed only to obtain translation δ_{YYo} needed for the calculation of angle a of the principal system.

The angle a of the principal system of diaphragm i is calculated by means of the equation C.9.1 of §C.9 using the 2nd and 3rd analysis results:

$tan(2a)=2\delta_{XYo}/(\delta_{XXo}-\delta_{YYo})=2\times(-0.047)/(2.103-1.668)=-0216 \rightarrow 2a=-12.2^o \rightarrow a=-6.1^o$

Step 4: Analysis results from loading 1 minus analysis results from loading 2, meaning that only moment and only rotation exists about the centre of stiffness C_T, which remains stationary with respect to the ground.

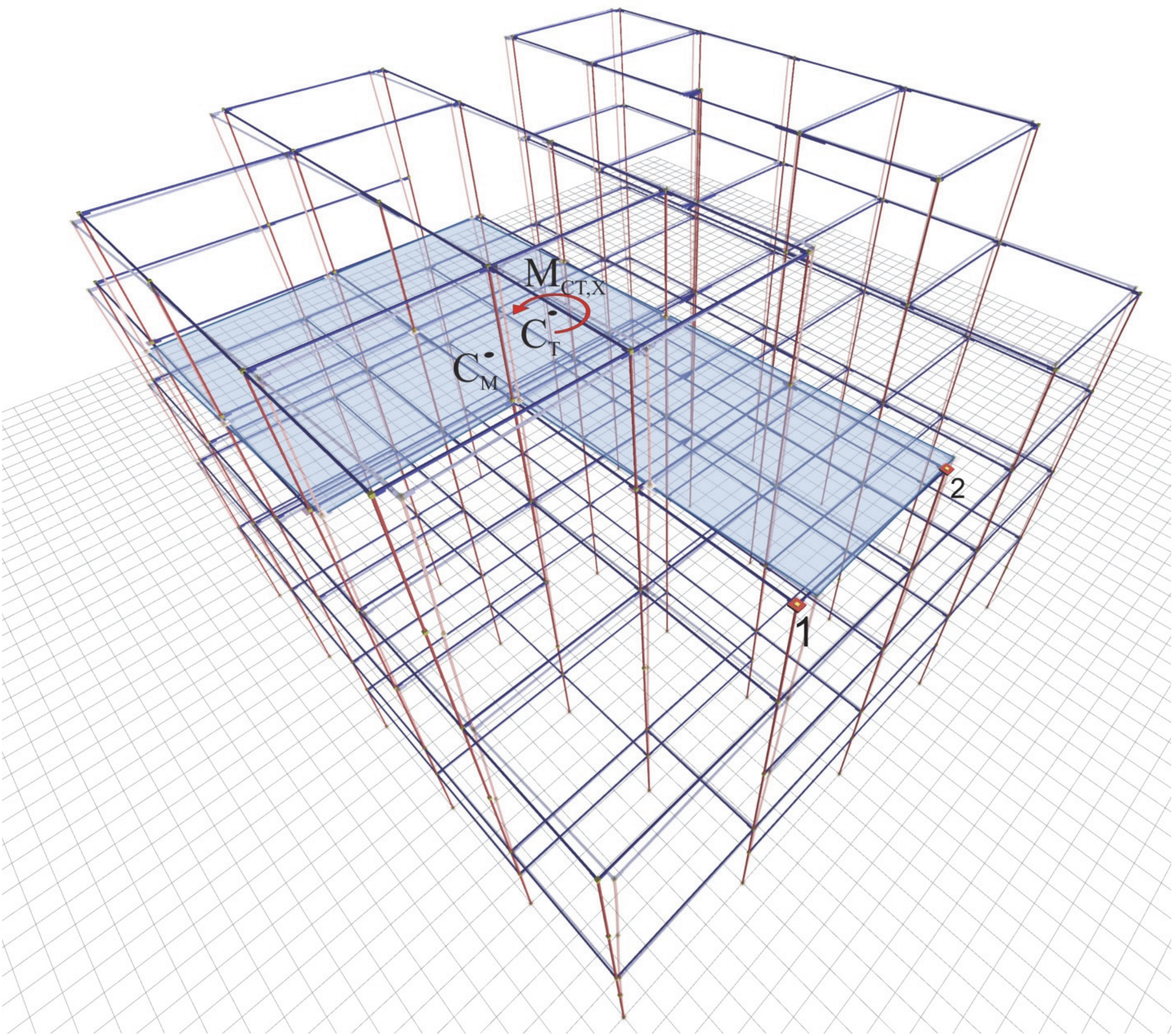

Figure D.2-7: The only load acting on diaphragm i is moment on C_T
Diaphragm displacements are induced only due to rotation
The centre of stiffness C_T remains stationary with respect to the ground

Using this trick, namely by subtracting analysis 2 results from analysis 1 results, the diaphragm i develops only rotation, while the remaining diaphragms develop both translations and rotation. However only diaphragm i is examined here. The most important result of this trick is that the diaphragm i is rotated about C_T, which remains stationary with respect to the ground, allowing the calculation of its precise location. The rotation angle of diaphragm i is the rotation angle θ_{XZ} calculated in step 1 under the corresponding loading.

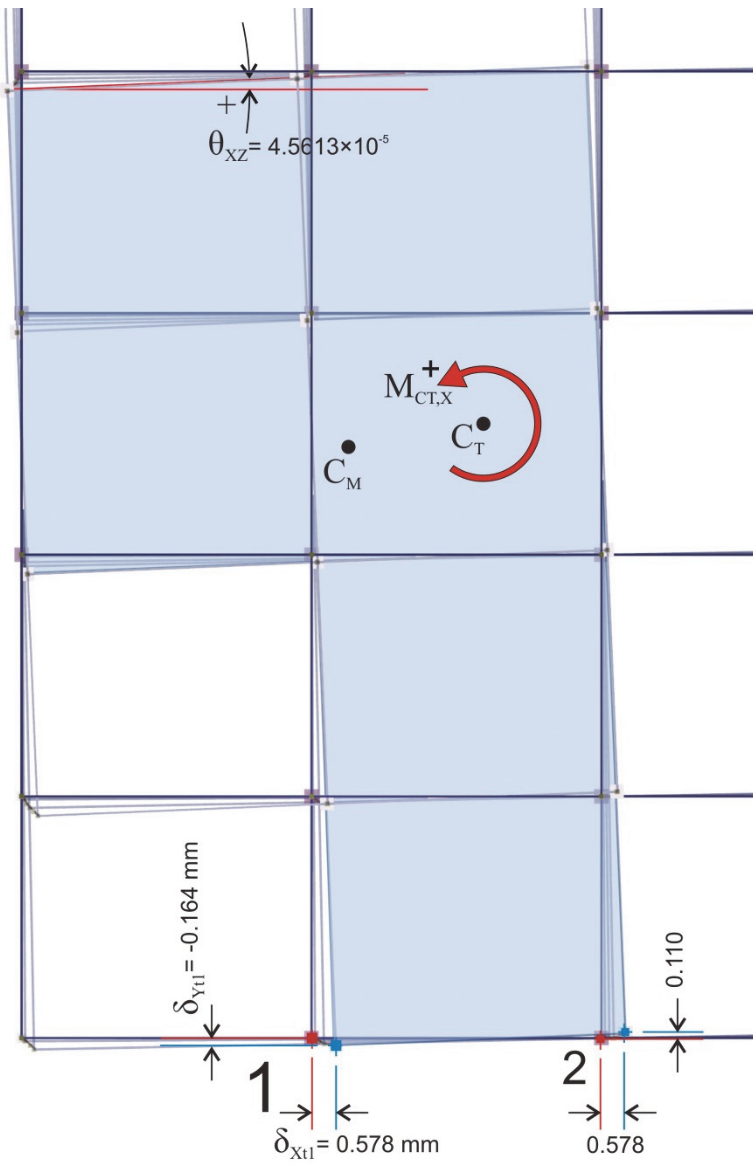

Figure D.2-8

The displacements of an arbitrary point j of the diaphragm, due to rotation θ_{XZ} derive form the expressions:

$\delta_{Xtj}=\delta_{XXj}-\delta_{XXo}$, $\delta_{Ytj}=\delta_{XYj}-\delta_{XYo}$

The C_T coordinates corresponding to these displacements are (see §C.5):

$\boldsymbol{X_{CT}=X_j-\delta_{Ytj}/\theta_{XZ}}$
$\boldsymbol{Y_{CT}=Y_j+\delta_{Xtj}/\theta_{XZ}}$

In the example considered, for point 1:
$\delta_{Xt1}=\delta_{XX1}-\delta_{XXo}=2.681-2.103=0.578$ mm
$\delta_{Yt1}=\delta_{XY1}-\delta_{XYo}=-0.211-(-0.047)=-0.164$ mm
The diaphragm rotation is identical to that of analysis 1, i.e. $\theta_{XZ}=4.5613\times10^{-5}$.
The coordinates of point 1 are (6.0, 0.0), thus:
$X_{CT}=X_1-\delta_{Yt1}/\theta_{XZ}=6.0-(-0.164)\times10^{-3}/(4.5613\times10^{-5})=9.6$ m
$Y_{CT}=Y_1+\delta_{Xt1}/\theta_{XZ}=0.0+0.578\times10^{-3}/(4.5613\times10^{-5})=12.7$ m

Note:

To verify the generality of the method, C_T coordinates are calculated from point 2 (12.0, 0.0) as well:

$\delta_{Xt2}=\delta_{XX2}-\delta_{XXo}=2.681-2.103=0.578$ mm

$\delta_{Yt2}=\delta_{XY2}-\delta_{XYo}=0.063-(-0.047)=0.110$ mm

$X_{CT}=X_2-\delta_{Yt2}/\theta_{XZ}=12.0-0.110\times10^{-3}/(4.5613\times10^{-5})=9.6$ m

$Y_{CT}=Y_2+\delta_{Xt2}/\theta_{XZ}=0.0+0.578\times10^{-3}/(4.5613\times10^{-5})=12.7$ m

In the same way, C_T coordinates may be verified using any point of the diaphragm.

D.3 Diaphragm lateral stiffnesses

Each diaphragm has two lateral stiffnesses in the two principal directions.
Lateral stiffness of diaphragm K_{xx} (or K_{yy}) along the principal x (or y) direction, is defined as the ratio of the seismic force H to the corresponding δ_{xo} (or δ_{yo}) displacement of the diaphragm, when force acts on C_T along the principal direction considered, i.e.:

$K_{xx}=H_{xx}/\delta_{xxo}$, $K_{yy}=H_{xy}/\delta_{xyo}$ or equivalent: $K_{xx}=H_{yx}/\delta_{yxo}$, $K_{yy}=H_{yy}/\delta_{yyo}$

Using analysis 2, the external force $H_X=100$ of the initial system X0Y is resolved into two equivalent principal forces $H_{xx}=100\cdot cosa$ and $H_{xy}=-100\cdot sina$ (see §C.2) acting in x,y directions of the principal system. In the present case, $a=-6.154^o$, therefore $H_{xx}=100\times 0.994=99.4\ kN$ and $H_{xy}=-100\times(-0.107)=10.7\ kN$.
Respectively, the two translations of the centre of stiffness in X0Y, derived from analysis 2, are $\delta_{XXo}=2.103$, $\delta_{XYo}=-0.047$, which transferred in the principal system yield (§C.2):

$\delta_{xxo}=\delta_{XXo}\cdot cosa-\delta_{XYo}\cdot sina=2.103\times 0.994-(-0.047)\times(-0.107)=2.090-0.005=2.085\ mm$

$\delta_{xyo}=-\delta_{XXo}\cdot sina+\delta_{XYo}\cdot cosa=-2.103\times(-0.107)+(-0.047)\times 0.994=0.225-0.047=0.178\ mm$, άρα

$K_{xx}=H_{xx}/\delta_{xxo}=99.4\times 10^3 N/2.085\times 10^{-3} m=47.7\times 10^6\ N/m$

$K_{yy}=H_{xy}/\delta_{xyo}=10.7\times 10^3 N/0.178\times 10^{-3} m=60.1\times 10^6\ N/m$

Notes:

- *If analysis 3 is usedthen:*

  $H_{yx}=100\cdot sina=-10.7\ kN$ *and* $H_{yy}=100\cdot cosa=99.4\ kN$

  $\delta_{yxo}=\delta_{YXo}\cdot cosa+\delta_{YYo}\cdot sina=(-0.047)\times 0.994+1.668\times(-0.107)=-0.047-0.178=0.225,$

  $\delta_{yyo}=-\delta_{YXo}\cdot sina+\delta_{YYo}\cdot cosa=-(-0.047)\times(-0.107)+1.668\times(0.994)=-0.005+1.658=1.653$

  $K_{xx}=H_{yx}/\delta_{yxo}=-10.7\times 10^3 N/(-0.225\times 10^{-3} m)=47.6\times 10^6\ N/m$

  $K_{yy}=H_{yy}/\delta_{yyo}=99.4\times 10^3 N/1.653\times 10^{-3} m=60.1\times 10^6\ N/m$

  Namely the same stiffness values are obtained, as expected.
- *The expressions C.9.2 and C.9.3 of §C.9 could be used for the calculation of stiffnesses* K_{xx}, K_{yy}, *therefore:*

  $K_{xx}=H/(\delta_{XXo}+\delta_{XYo}\cdot tana)=100.0/(2.103-0.047\times 0.107)\times 10^6 N/m=47.6\times 10^6\ N/m$

  $K_{yy}=H/(\delta_{YYo}-\delta_{XYo}\cdot tana)=100.0/(1.668+0.047\times 0.107)\times 10^6 N/m=60.0\times 10^6\ N/m$
- *In case the centres of mass and stiffness coincide,* K_{xx} *should be calculated from analysis 2, while* K_{yy} *from analysis 3, avoiding division by zero.*

D.4 Diaphragm torsional stiffness

Torsional stiffness of diaphragm K_θ is defined as the ratio of moment M_{CT} acting on the centre of stiffness C_T, to the rotation angle θ_z of the diaphragm.
Moment acting on C_T is equal to

$M_{XCT}=100.0\cdot(Y_{CT}-Y_{CM})+100.0\cdot c_Y=100\times(12.685-12.323)+200.0=36.2+200.0=236.2\ kNm$

$K_\theta=M_{XCT}/\theta_{XZ}=236.2\times 10^3 Nm/4.5613\times 10^{-5}=5178\times 10^6\ Nm$

Note:

Moment M_{XCT} *of external forces is the sum of* $M_{XH}=36.2\ kNm$ *due to external force H and the external moment* $M_{XM}=200.0\ kNm$. *Respectively, the rotation angle* θ_{XZ} *of the diaphragm is the sum of*

$\theta_{XZH}=(M_{XH}/M_{XCT})\cdot\theta_{XZ}=(36.2/236.2)\times 4.5613\times 10^{-5}=0.699\times 10^{-5}$ *and*

$\theta_{XZM}=(M_{XM}/M_{XCT})\cdot\theta_{XZ}=(200/236.2)\times 4.5613\times 10^{-5}=3.8622\times 10^{-5}$

Also $K_\theta=M_{XH}/\theta_{XZH}=M_{XM}/\theta_{XZM}=5178\times10^6$ Nm.

The constant moment value in all diaphragms and the respective calculation of torsional stiffness is utilized in the determination of the equivalent system of §D.6.

D.5 Torsional stiffness distribution

Torsional stiffness distribution of diaphragm is the curve on which, if the idealised columns with the same lateral stiffnesses as those of the diaphragm, are placed symmetrically with respect of the centre of stiffness, the torsional stiffness derived is the same as that of the diaphragm.

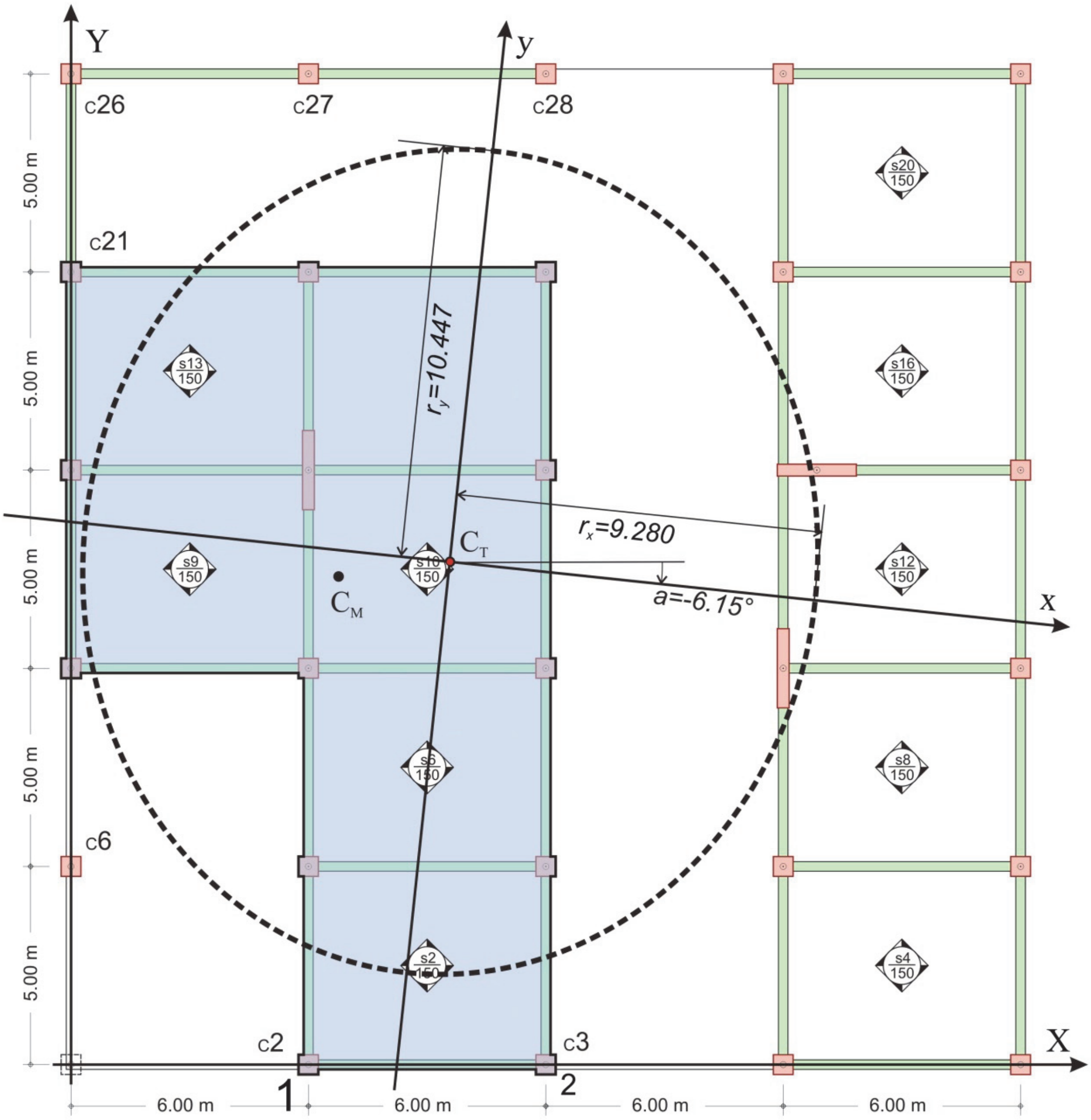

Figure D.5

Torsional stiffness ellipse of diaphragm is defined as the ellipse having C_T as centre and $r_x=\sqrt{(K_\theta/K_{yy})}$, $r_y=\sqrt{(K_\theta/K_{xx})}$ as radii.

$r_x=\sqrt{K_\theta/K_{yy}}=\sqrt{[5178\times10^6 Nm/60.1\times10^6 N/m]}=9.28\ m$
$r_y=\sqrt{K_\theta/K_{xx}}=\sqrt{[5178\times10^6 Nm/47.7\times10^6 N/m]}=10.42\ m$

D.6 Equivalent system – Relative lateral stiffness – Relative torsional stiffness

A simple, yet equivalent building is created on the basis of appendix B', §B.1 and §B.2 and the previous paragraphs of the present appendix. The equivalent building should comprise same number of floors and diaphragms. The columns of each floor should have specific dimensions, thus for each floor relative lateral and torsional stiffnesses are used (see §5.4.3.5 and §5.4.4).

Each diaphragm is replaced by an equivalent one consisting of *4* fixed-ended columns placed symmetrically with respect to its centre of stiffness.

The equivalent diaphragm *i* is assumed to behave as an one-storey structure of fixed-ended columns. The four relative translations of the centre of stiffness along *X, Y* axes $\delta_{XXoZ,i}$, $\delta_{XYoZ,i}$, $\delta_{YXoZ,i}$, $\delta_{YYoZ,i}$ [1] as well as the rotation angle $\theta_{XZMZ,I}$ are defined as functions of the actual building displacements as follows

$\delta_{XXoZ,i}= \delta_{XXo,i}-\delta_{XXo,i-1}$, $\delta_{XYoZ,i}=\delta_{XYo,i}-\delta_{XYo,i-1}$,
$\delta_{YXoZ,i}= \delta_{YXo,i}-\delta_{YXo,i-1}$ [2], $\delta_{YYoZ,i}=\delta_{YYo,i}-\delta_{YYo,i-1}$,
$\theta_{XZMZ,i} = \theta_{XZM,i} - \theta_{XZM,i-1}$ [3].

Therefore, the data of the equivalent diaphragm are: rotation angle $a_{z,i}=2\delta_{XYoZ,i}/(\delta_{XXoZ,i}-\delta_{YYoZ,i})$, torsional stiffness $K_{\theta Z,i}=H\cdot c_Y/\theta_{XZMZ,i}$, lateral stiffnesses $K_{xxZ,i}=H/(\delta_{XXoZ,i}+\delta_{XYoZ,i}\cdot tana_{Z,i})$ and $K_{yyZ,i}=H/(\delta_{YYoZ,i}-\delta_{XYoZ,i}\cdot tana_{Z,i})$ *(expressions C.9.2 and C.9.3 of §C.9).*
Also $r_{xZ,i}=\sqrt{K_{\theta Z,i}/K_{yyZ,i}}$, $r_{yZ,i}=\sqrt{K_{\theta Z,i}/K_{xxZ,i}}$.

The location of the equivalent diaphragms is determined by translating the centre of stiffness to point 0.0, 0.0. In this way the equivalent building is created and for its diaphragms i the following relations apply:

$\delta_{XXo\_equal,i}=\Sigma(\delta_{XXoZ,k})$ όπου $k=i$ έως 1 → $\delta_{XXo\_equal,i}=\Sigma(\delta_{XXo,i}-\delta_{XXo,i-1})=(\delta_{XXo,i}-\delta_{XXo,i-1})+(\delta_{XXo,i-1}-\delta_{XXo,i-2})+\ldots+(\delta_{XXo,1}-0.0)=\delta_{XXo,i}$ → $\delta_{XXo\_equal,i}=\delta_{XXo,i}$
Using the same logic $\delta_{XYo\_equal,i}=\delta_{XYo,i}$, $\delta_{YYo\_equal,i}=\delta_{YYo,i}$ *and* $\theta_{XZ\_equal,i}=\theta_{XZM,i}$
Therefore all diaphragm quantities $K_{xx\_equal,i}$, $K_{yy\_equal,i}$, $K_{\theta\_equal,i}$ are equal to those of the actual building.

Two analytical examples for the application of the method are presented in the following paragraph.

The seismic assessment of the actual building can be performed in an easy, direct and descriptive way by means of the equivalent building.

<u>Notes:</u>

- *In ground floor diaphragms, their torsional stiffness ellipse coincides with the ellipse of the corresponding equivalent diaphragms.*
- $1/K_{\theta\_equal,i}=\Sigma(1/K_{\theta Z,j})$, $1<j\leq i$

[1] Subscript *z*, before, i denotes that the value of the quantity considered is relative rather than absolute.

[2] Always $d_{XYoZ,i}=d_{YXoZ,i}$ due to $\delta_{XYo,i}=\delta_{YXo,i}$ and $\delta_{XYo,i-1}=\delta_{YXo,i-1}$, thus the same is valid for their difference.

[3] Subscript *M* implies rotation angle of the diaphragm due to moment $M_M=H\cdot c$ as described in the note of §D.4.

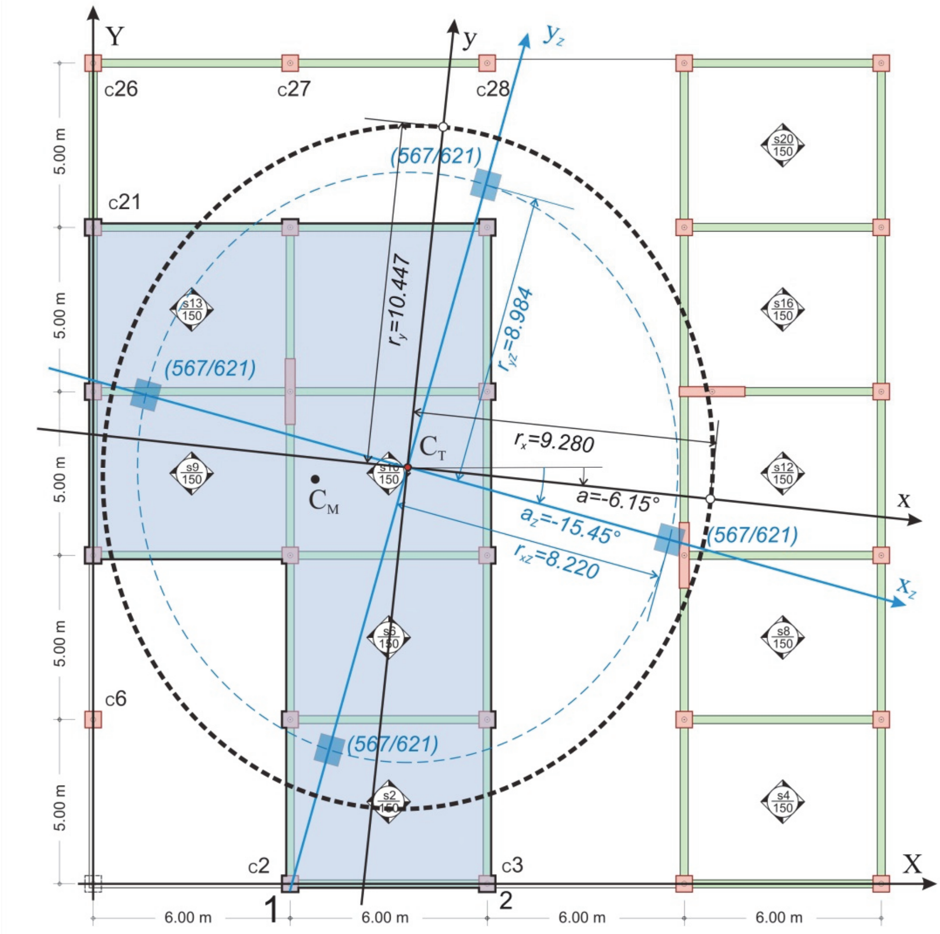

Figure D.6: The equivalent diaphragm of the diaphragm on the left of 4th floor

(n=4 equivalent columns of 567/621 cross-section)