## 4.5 One-way slabs

One-way slabs are those supported on two opposite edges, such as slab s1 in figure of §4.1.

If a one-way slab is supported on more than two edges and its aspect ratio, i.e. the ratio of the larger to the smaller theoretical span, is greater than 2.0 (such as slab s3 in the same figure), it is considered as one-way slab in the principal direction while taking into account the secondary stresses in rest edges.

### 4.5.1 Static analysis

Continuous one-way slabs are analysed considering a frame of continuous member s of rectangular shape cross-section, having width equal to 1.00 m and height equal to the thickness of the slab. The strip loads comprise self-weight, dead and live loads.

Analysis is performed:

α) approximately, by applying all design loads p=1.35g+1.50q (when live load is relatively small)

β) accurately, by taking into account unfavourable loadings.

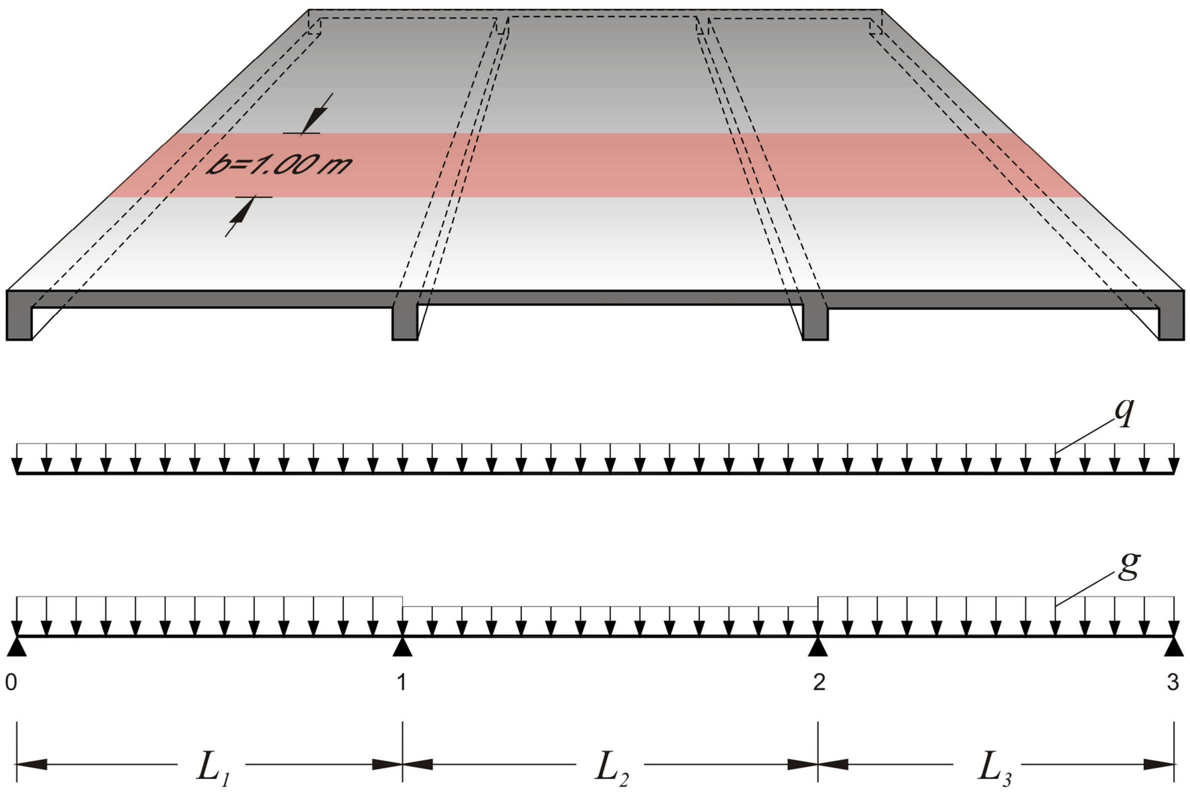


*Figure 4.5.1-1: Three-span continuous slab*

*<u>Example:</u>*

*The three slabs (previous figure) have $L_1$=4.50 m, $h_1$=180 mm, $g_1$=10.0 kN/m$^2$, $q_1$=2.0 kN/m$^2$, $L_2$=4.00 m, $h_2$=140 mm, $g_2$=5.0 kN/m$^2$, $q_2$=2.0 kN/m$^2$, $L_3$=4.00 m, $h_3$=140 mm, $g_3$=5.0 kN/m$^2$, $q_3$=2.0 kN/m$^2$, where loads g include self-weight. Perform static analysis considering global loading in ultimate limit state.*

*Design load in each slab is equal to $p_i=\gamma_g \cdot g_i+\gamma_q \cdot q_i=1.35 \cdot g_i+1.50 \cdot q_i$, thus on 1.00 m wide strip, it is:*

$p_1=1.35\times10.0+1.50\times2.0=16.5$ kN/m

$p_2=p_3=1.35\times5.0+1.50\times2.0=9.75$ kN/m

*The three-span continuous slab will be solved through Cross method.*

*Fundamental design span moments (table b3)*

$M_{10}=-p_1 \cdot L_1^2/8=-16.5\times4.50^2/8=-41.8$ kNm

$M_{12}=M_{21}=-p_2 \cdot L_2^2/12=-9.75\times4.00^2/12=-13.0$ kNm

$M_{23}=-p_3 \cdot L_3^2/8=-9.75\times4.00^2/8=-19.5$ kNm

*Moments of inertia I*

$I_{01}=I_c=1.0\times0.18^3/12=4.86\times10^{-4}$ m$^4$

$I_{12}=I_{23}=1.0\times0.14^3/12=2.29\times10^{-4}$ m$^4$=0.47$I_c$

*Stiffness factors k, distribution indices υ*

$k_{10}=\frac{3I_{10}}{4I_c \cdot L_{01}}=\frac{3}{4\times4.5}=$ 0.167 $\quad \upsilon_{01}=\frac{0.167}{0.285}$ 0.586

$k_{12}=\frac{4I_{12}}{4I_c \cdot L_{12}}=\frac{4\times0.47I_c}{4I_c\times4.0}=$ 0.118 $\quad \upsilon_{12}=\frac{0.118}{0.285}$ 0.414

0.285 $\quad$ 1.000

$k_{21}=k_{12}=$ 0.118 $\quad \upsilon_{21}=\frac{0.118}{0.206}$ 0.573

$k_{23}=\frac{3I_{23}}{4I_c \cdot L_{23}}=\frac{3\times0.47I_c}{4I_c\times4.0}=$ 0.088 $\quad \upsilon_{01}=\frac{0.088}{0.206}$ 0.427

0.206 $\quad$ 1.000

| | 1 | | 2 | |
|---|---|---|---|---|
| 0.586 | 0.414 | | 0.573 | 0.427 |
| +41.8 | -13.0 | | +13.0 | -19.5 |
| -[+41.8-13.0]×0.586→ - 16.9 | -11.9 | →0.50 | - 6.0 | |
| | + 3.6 | 0.50← | + 7.2 | + 5.3 ← 0.427×[-(+13.0-19.5-6.0)] |
| -[+3.6]×0.586→ - 2.1 | - 1.5 | →0.50 | - 0.8 | |
| | + 0.3 | 0.50← | + 0.5 | + 0.3 ← 0.427×[-(-0.8)] |
| -[+0.3]×0.586→ - 0.2 | - 0.1 | | | |
| +22.6 | -22.6 | | +13.9 | -13.9 |

$M_1$=-22.6 kNm $\qquad$ $M_2$=-13.9 kNm

$V_{01}$=16.5×4.50/2-22.6/4.50=32.1 kN

$V_{10}$=-16.5×4.50/2-22.6/4.50=-42.1 kN

$V_{12}$=9.75×4.00/2+(-13.9+22.6)/4.00=21.7 kN

$V_{21}$=-9.75×4.00/2+(-13.9+22.6)/4.00=-17.3 kN

$V_{23}$=9.75×4.00/2+13.9/4.00=23.0 kN

$V_{32}$=-9.75×4.00/2+13.9/4.00=-16.0 kN

$maxM_{01}=32.1^2/(2×16.5)=31.2$ kNm

$maxM_{12}=21.7^2/(2×9.75)-22.6=1.5$ kNm

$maxM_{23}=16.0^2/(2×9.75)=13.1$ kNm

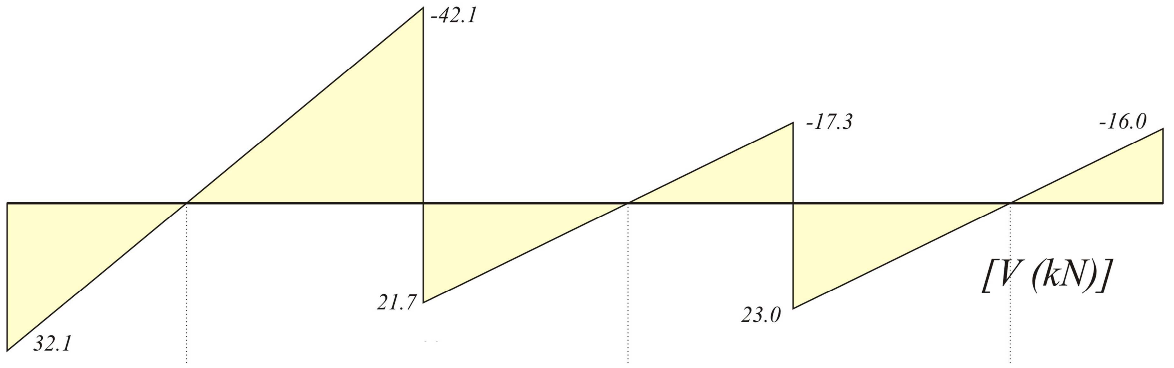


Figure 4.5.1-2: Shear force diagram

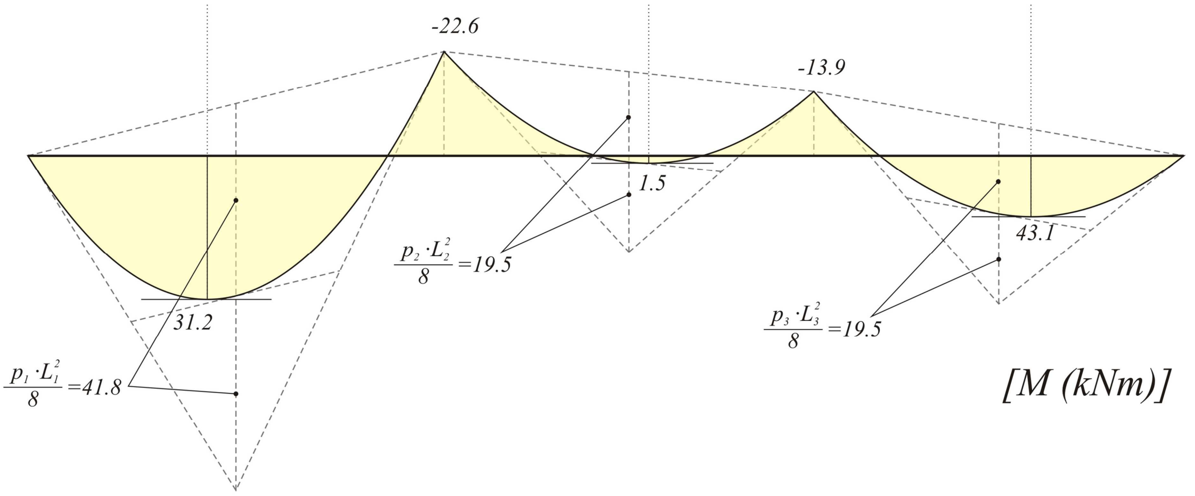


Figure 4.5.1-3: Bending moment diagram

### 4.5.2 Deflection

Slab member AB of length $L$, moment of inertia $I$, elasticity modulus $E$, is subjected to uniform load $p$. Given shear force $V_{A,R}$ (at left support) and bending moment $M_A$, calculate equation of elastic line due to bending and maximum deflection.

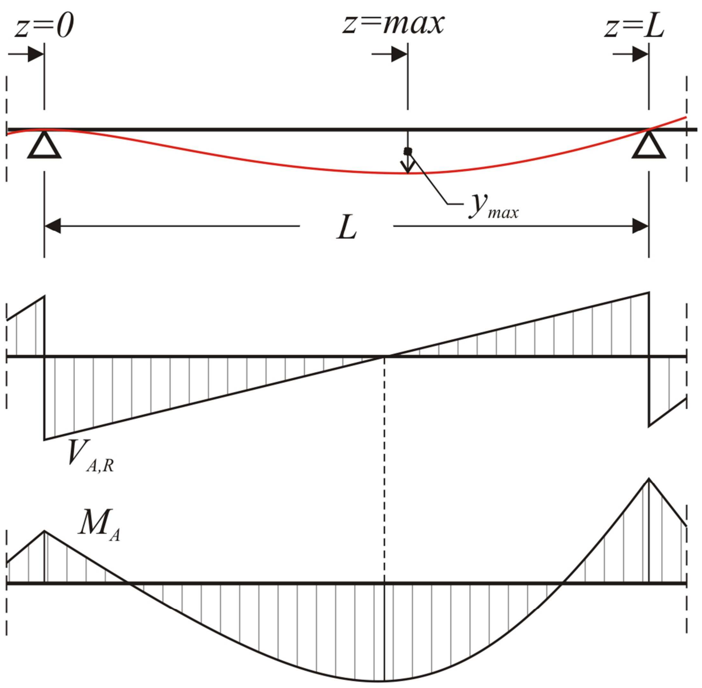


*Figure 4.5.2-1: General case of bending of member (slab or beam)*

Considering coordinate z origin at the left end:

$$V(z) = V_{A,R} - p \cdot z$$

$$M(z) = M_A + V_{A,R} \cdot z - \frac{p \cdot z^2}{2}$$

The basic equation of elastic line $E \cdot I \cdot \frac{d^2 y(z)}{dz^2} = -M(z)$ is solved in two steps:

<u>Step 1</u>

$$\varphi(z) = \frac{dy(z)}{dz} = \frac{1}{E \cdot I} \cdot \int -M(z)dz = \frac{1}{E \cdot I} \cdot \int (-M_A - V_{A,R} \cdot z + \frac{p \cdot z^2}{2})dz \rightarrow$$

$$\varphi(z) = \frac{1}{E \cdot I} \cdot (-M_A \cdot z - \frac{V_{A,R} \cdot z^2}{2} + \frac{p \cdot z^3}{6} + C_1)$$

Hence, the equation of the elastic line tangents is:

$$\varphi(z) = \frac{1}{E \cdot I} \cdot (\frac{p}{6} \cdot z^3 - \frac{V_{A,R}}{2} \cdot z^2 - M_A \cdot z + C_1) \quad (1)$$

Step 2

$$y(z)=\int \varphi(z)dz=\frac{1}{E\cdot I}\cdot\int\left(\frac{p}{6}\cdot z^3-\frac{V_{A,R}}{2}\cdot z^2-M_A\cdot z+C_1\right)dz\rightarrow$$

$$y(z)=\frac{1}{E\cdot I}\cdot\left(\frac{p}{24}\cdot z^4-\frac{V_{A,R}}{6}\cdot z^3-\frac{M_A}{2}\cdot z^2+C_1\cdot z+C_2\right)$$

$y(0)=0 \rightarrow C_2=0$

Hence, the equation of the elastic line is:

$$y(z)=\frac{1}{E\cdot I}\cdot\left(\frac{p}{24}\cdot z^4-\frac{V_{A,R}}{6}\cdot z^3-\frac{M_A}{2}\cdot z^2+C_1\cdot z\right)\quad(2)$$

$y(L)=0 \rightarrow$

$$0=\frac{1}{E\cdot I}\cdot\left(\frac{p\cdot L^4}{24}-\frac{V_{A,R}\cdot L^3}{6}-\frac{M_A\cdot L^2}{2}+C_1\cdot L\right)\rightarrow C_1=-\frac{p\cdot L^3}{24}+\frac{V_{A,R}\cdot L^2}{6}+\frac{M_A\cdot L}{2}\quad(3)$$

Thus, the equations of the elastic line tangents (1) and deflections (2) are determined.

The maximum deflection is at the location where the first derivative of the elastic line equation is zero, i.e. at the point z where φ(z) =0.

(1) $\rightarrow \frac{p\cdot z^3}{6}-\frac{V_{A,R}\cdot z^2}{2}-M_A\cdot z+C_1=0$ (4)

The real positive root of the cubic equation (3) gives the desired point $z_{max}$, which replaced in equation (2) yields the maximum deflection $y_{max}$.

*<u>Example:</u>* ***Deflection of first slab*** *(example of §4.3.1):*

*For L=4.5 m, p=16.5 kN/m,* $V_{A,R}$*=32.1 kN and* $M_A$*=0.0, expression (3) yields:*

$$C_1=-\frac{16.5\times4.5^3}{24}+\frac{32.1\times4.5^2}{6}kN\cdot m^2=45.7\ kN\cdot m^2$$

(4) $\rightarrow (16.5/6)\cdot z^3-(32.1/2)\cdot z^2-0+45.7=0 \rightarrow 2.75z^3-16.05z^2+45.7=0 \rightarrow z_{max}=2.112\ m$

(2) $\rightarrow y(z)=\frac{1}{E\cdot I}\cdot(0.6875\cdot z^4-5.35\cdot z^3+45.7\cdot z)$ (1.2)

$$y(2.112)=\frac{1}{E\cdot I}\cdot(0.6875\times2.112^4-5.35\times2.112^3+45.7\times2.112)\cdot10^3N\cdot m^3=\frac{59.8}{E\cdot I}\cdot10^3N\cdot m^3$$

*For slab thickness h=180 mm and modulus of elasticity for concrete E=32.80 GPa:*

$I=(b\cdot h^3)/12=(1.0\times0.18^3)/12=486\times10^{-6}\ m^4$

$E\cdot I=32.8\times10^9N/m^2\times486\times10^{-6}m^4=15.9408\times10^6\ N\cdot m^2$, *therefore,*

$$y_{1,max}=y(2.112)=\frac{59.8\cdot10^3N\cdot m^3}{15.9408\cdot10^6N\cdot m^2}=0.00375\ m=3.75\ mm$$

The elastic line of the continuous slab given by expressions (1.2), (2.2), (3.2) is illustrated in the following figure:

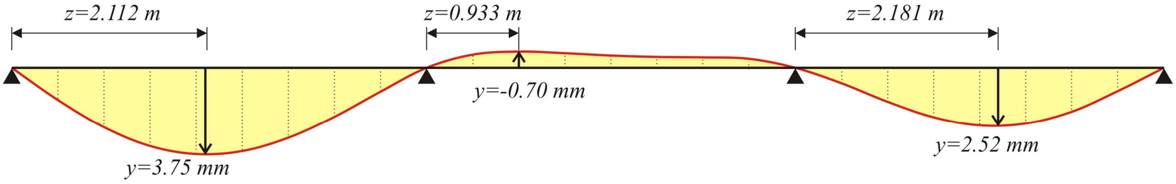


*Figure 4.5.2-2: The elastic line of the three slabs (from the equations)*

Project <B_451> (pi-FES) produces identical deflections:

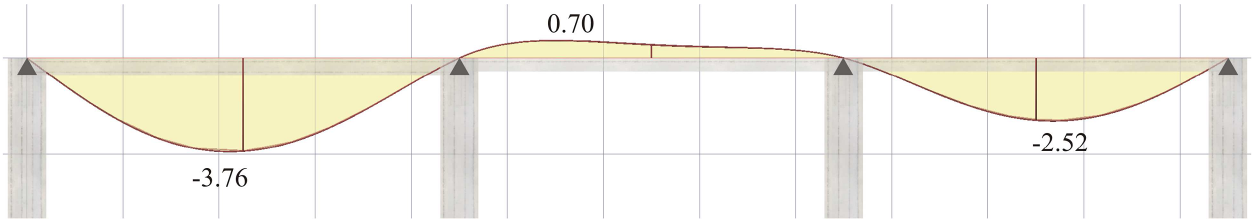


*Figure 4.5.2-3: Front view of the elastic line (from pi-FES with active module\SLABS)*

**Maximum support moments** (void loading on adjacent spans and alternating with the rest)

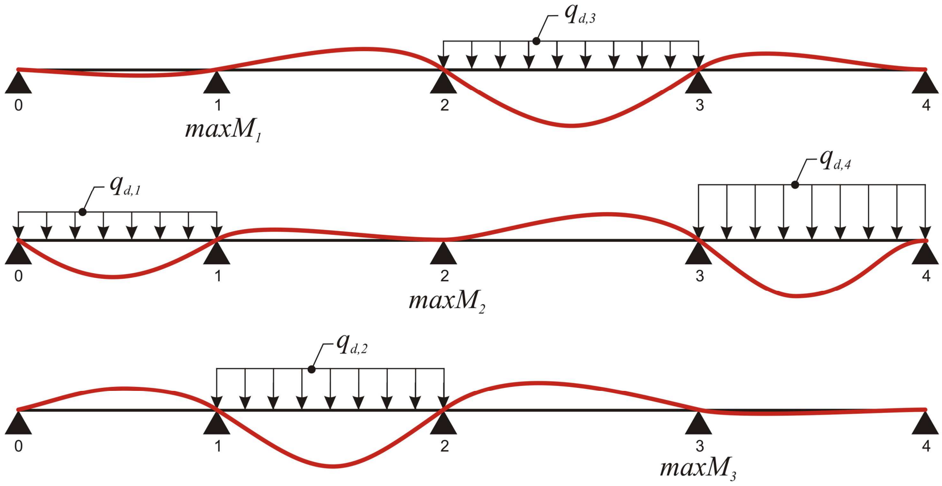


Figure 4.5.3.1-4

<u>*Example:*</u>

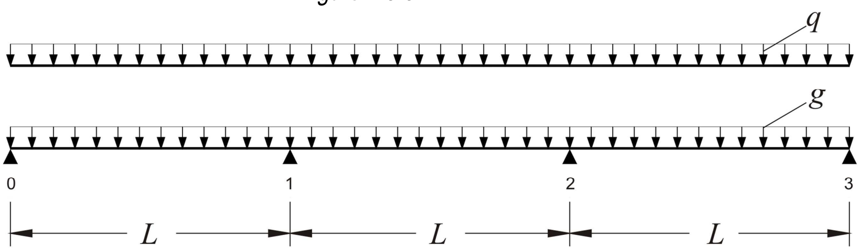


Figure 4.5.3.1-5

*The continuous slab shown in the figure, of span length L=5. 00 m and of thickness h=160 mm, is subjected to covering load $g_e$=1.0 kN/m$^2$ and live load q=5.0 kN/m$^2$. Concrete class C50/60. Calculate the shear forces and bending moments envelopes for the three slabs, in ultimate limit state.*

*Solution:*

| | | | | |
|---|---|---|---|---|
| *Self-weight:* | $g_o$ | = | $0.16m \cdot 25.0kN/m^3$ = | 4.00 kN/m$^2$ |
| *Covering load:* | $g_e$ | = | | 1.00 kN/m$^2$ |
| *Total dead loads:* | $g$ | = | | 5.00 kN/m$^2$ |
| *Total live loads:* | $q$ | = | 5.00 kN/m$^2$ | |

*The design dead load for each slab is $g_d$=1.00×5.0=5.0 kN/m and the total design load is $p_d=\gamma_g \cdot g+\gamma_q \cdot q$=1.35×5.0+1.50×5.0=14.25 kN/m.*

*Manual calculations:* $I=(b \cdot h^3)/12=(1.0\times0.16^3)/12=341\times10^{-6}\ m^4$

*Modulus of elasticity for concrete C50/60 is equal to E=37.3 GPa.*

$E \cdot I=37.3\times10^9 N/m^2\times341\times10^{-6}m^4=12.719\times10^6\ N \cdot m^2$

*For $I_{10}=I_{12}=I_{23}=I_c$, stiffness factors k distribution indices υ are:*

$$k_{10}=\frac{3I_{10}}{4I_c \cdot L_{01}}=\frac{3}{4\times5.0}= \quad 0.150 \qquad \upsilon_{01}=\frac{0.150}{0.350}= \quad 0.429$$

$$k_{12}=\frac{4I_{12}}{4I_c \cdot L_{12}}=\frac{4}{4\times5.0}= \quad \underline{0.200} \qquad \upsilon_{12}=\frac{0.200}{0.350}= \quad \underline{0.571}$$

$$0.350 \qquad\qquad 1.000$$

*Due to the symmetry of the structure :* $\upsilon_{21}=0.571$ *και* $\upsilon_{23}=0.429$

*Loading 1*: $w_1=w_3=p_d=14.25$ kN/m, $w_2=g_d=5.0$ kN/m ($V_{01,max}$, $M_{01,max}$, $M_{12,min}$, $|V_{32,max}|$, $M_{23,max}$)

*Principal support moments from table b3* →

$M_{10}=M_{23}=-w_1 \cdot L^2/8=-14.25\times5.0^2/8=-44.5$ kNm, $M_{12}=M_{21}=-w_2 \cdot L^2/12=-5.0\times5.0^2/12=-10.4$ kNm

| 0.429 | 0.571 | | 0.571 | 0.429 |
|---|---|---|---|---|
| +44.5 | -10.4 | | +10.4 | -44.5 |
| -[+44.5-10.4]×0.429→ -14.6 | -19.5 | →0.50 | - 9.8 | |
| | +12.5 | 0.50← | + 25.1 | +18.8← 0.429×[-(+10.4-44.5-9.8)] |
| -[+12.5×0.429]→ -5.3 | - 7.2 | →0.50 | - 3.6 | |
| | + 1.1 | 0.50← | + 2.1 | + 1.5 ← 0.429×[-(-3.6)] |
| -[+1.1×0.586]→ -0.6 | - 0.5 | →0.50 | -0.3 | |
| | | | +0.2 | + 0.1 ← 0.429×[-(-0.3)] |
| +24.1 | -24.1 | | +24.1 | -24.1 |
| $M_1=-24.1$ kNm | | | $M_2=-24.1$ kNm | |

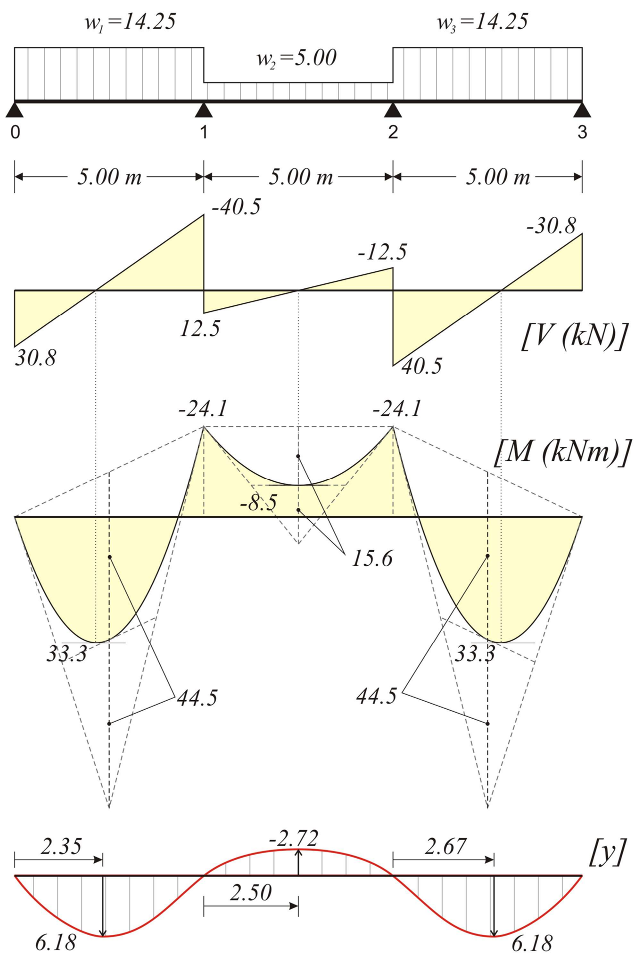


Figure 4.5.3.1-6

$V_{01}=14.25\times5.0/2-24.1/5.0=35.63-4.82=30.8$ kN

$V_{10}=-35.63-4.82=-40.5$ kN

$V_{12}=5.0\times5.0/2=12.5$ kN

$M_{01,max}=V_{01}^2/(2 \cdot w_1)=30.8^2/(2\times14.25)=33.3$ kNm

$w_1 \cdot L^2/8=14.25\times5.0^2/8=44.5$ kNm

$M_{12,min}=V_{12}^2/(2 \cdot w_2)+M_1=12.5^2/(2\times5.0)-24.1=15.6-24.1=$

$=-8.5$ kN [12]

$w_2 \cdot L^2/8=5.0\times5.0^2/8=15.6$ kNm

01: (3) →

$C_1=(-14.25\times5.0^3/24+30.8\times5.0^2/6)=54.1$ kN·m$^2$

(4) → $(14.25/6)z^3-(30.8/2)z^2-0+54.1=0$ →

$2.375z^3-15.4z^2+54.1=0$ gives $z_{max}=2.347$ m

(2) → $y(z)=1/12.719\times[(14.25/24)\times2.347^4-(30.8/6)\times2.347^3+0\times2.347^2+54.1\times2.347)]$ →
$y(2.335)=6.18$ mm

12: Due to symmetry of both structure and loading
$z_{max}=2.5$ 0m

$C_1=(-5.00\times5.0^3/24+12.5\times5.0^2/6-24.1\times5.0/2)$kN·m$^2$=
$=-34.2$ kN·m$^2$

(2) → $y(z)=1/12.719\times[(5.00/24)\times2.50^4-(12.5/6)\times2.50^3+24.1\times2.50^2/2-34.2\times2.50)$ → $y(2.50)=-2.72$ mm

[12] The alternative calculation is: $M_{12}=w_1 \cdot L^2/8+M_1=5.0 \cdot 5.0^2/8-24.1=15.6-24.1=-8.5$ kNm

*Loading 2*: $w_1=w_3=g_d=5.0$ kN/m, $w_2=p_d=14.25$ kN/m ($V_{01,min}$, $M_{01,min}$, $M_{23,max}$, $|V_{32,min}|$, $M_{23,min}$)

*Principal support moments from table b3 →*

$M_{10}=M_{23}=-w_1 \cdot L^2/8=-5\times5.0^2/8=-15.6$ kNm, $M_{12}=M_{21}=-w_2 \cdot L^2/12=-14.25\times5.0^2/12=-29.7$ kNm

| 0.429 | 0.571 | | 0.571 | 0.429 |
|---|---|---|---|---|
| +15.6 | -29.7 | | +29.7 | -15.6 |
| -[+15.6-29.7]×0.429→ +6.1 | + 8.0 | →0.50 | +4.0 | |
| | - 5.2 | 0.50← | -10.3 | -7.8←0.429×[-(+29.7-15.6+4.0)] |
| -[-5.2]×0.429→ +2.2 | + 3.0 | →0.50 | +1.5 | |
| | - 0.5 | 0.50← | -0.9 | -0.6 ← 0.429×[-(+1.5)] |
| -[+0.5]×0.586→ + 0.2 | + 0.3 | →0.50 | +0.2 | |
| | | | - 0.1 | - 0.1 ← 0.429×[-(+0.2)] |
| +24.1 | -24.1 | | +24.1 | -24.1 |
| $M_1$=-24.1 kNm | | | $M_2$=-24.1 kNm | |

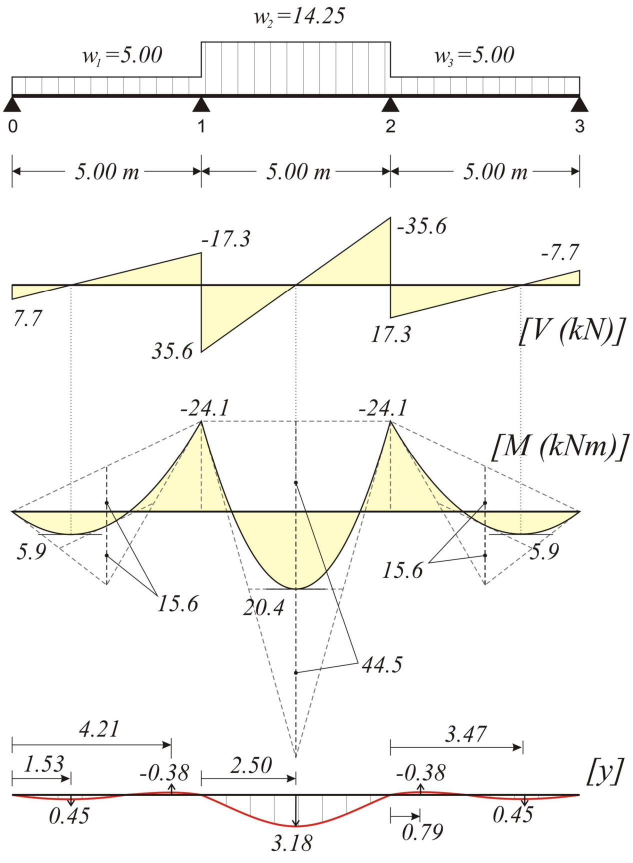


Figure 4.5.3.1-7

$V_{01}=5.0\times5.0/2-24.1/5.0=12.5-4.8=7.7$ kN

$V_{10}=-12.5-4.8=-17.3$ kN

$V_{12}=14.25\times5.0/2=35.6$ kN

$M_{01,max}=V_{01}^2/(2\cdot w_1)=7.7^2/(2\times5.0)=5.9$ kNm

$w_1\cdot L^2/8=5\times5.0^2/8=15.6$ kNm

$M_{12,max}=V_{12}^2/(2\cdot w_2)+M_1=35.6^2/(2\times14.25)-24.1=44.5-24.1=20.4$ kNm

$w_2\cdot L^2/8=14.25\times5.0^2/8=44.5$ kNm

01: (3) →

$C_1=(-5.00\times5.0^3/24+7.7\times5.0^2/6)$kN·m² $=6.0$ kN·m²

(4) → $(5.00/6)z^3-(7.7/2)z^2-0+6.0=0$ →

$0.833z^3-3.85z^2+6.0=0$ gives $z_{max,1}=1.53$ m και $z_{max,2}=4.21$ m

(2) → $y(z1)=1/12.719\times[(5.00/24)\times1.53^4-(7.7/6)\times1.53^3+0\times1.53^2+6.0\times1.53)$ → $y(1.53)=0.45$ mm

(2) → $y(z2)=1/12.719\times[(5.00/24)\times4.21^4-(7.7/6)\times4.21^3+0\times4.21^2+6.0\times4.21)$ → $y(4.21)=-0.39$ mm

12: Due to symmetry of both structure and loading $z_{max}=2.50$ m

$C_1=(-14.25\times5.0^3/24+35.6\times5.0^2/6-24.1\times5.0/2)$kN·m² $=$
$=13.9$ kN·m²

(2) → $y(z)=1/12.719\times[(14.25/24)\times2.50^4-(35.6/6)\times2.50^3+24.1\times2.50^2/2+13.9\times2.50)$ → $y(2.50)=3.18$ mm

*Loading 4: $w_1=g_d=5.0$ kN/m $w_2=w_3=p_d=14.25$ kN/m*

*This loading is an antisymmetric case of loading 3 with respect to the middle.*

*Envelopes of the results of all loadings:*

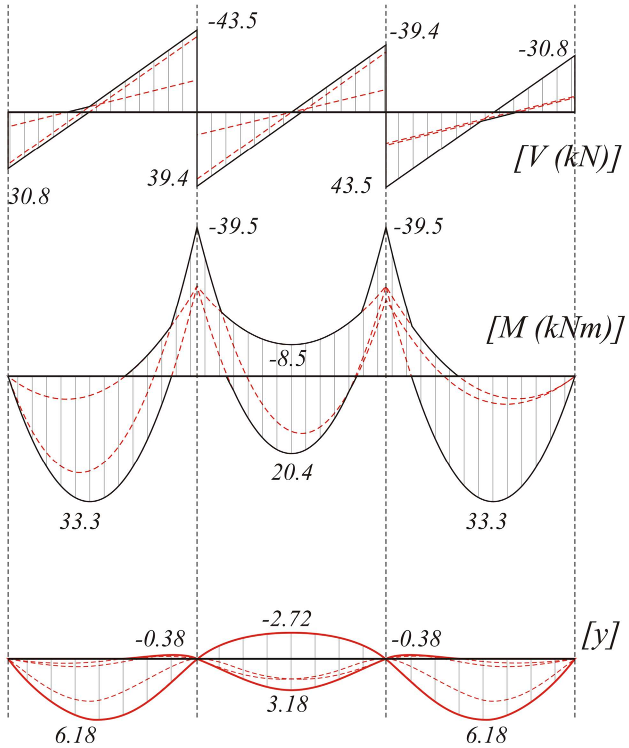


*Figure 4.5.3.1-9: Envelopes of shear forces – bending moments - deflections*

***Analysis using table b4***

$g_d/p_d=5.0/14.25=0.35$

$m_1=10.695$, $m_B=-9.025$, $m_2=17.425$, $p_{1A}=2.315$, $p_{1B}=-1.635$, $p_{2B}=1.805$

$V_{01,max}=p_d \cdot L/p_{1A}=14.25\times5.0/2.315=30.8$ kN

$V_{10,min}=p_d \cdot L/p_{1B}=-14.25\times5.0/1.635=-43.6$ kN

$V_{12,max}=p_d \cdot L/p_{2B}=14.25\times5.0/1.805=39.5$ kN

$M_{01,max}=p_d \cdot L^2/m_1=14.25\times5.0^2/10.695=33.3$ kNm

$M_{1,min}=p_d \cdot L^2/m_B=-14.25\times5.0^2/9.025=-39.5$ kNm

$M_{12,max}=p_d \cdot L^2/m_2=14.25\times5.0^2/17.425=20.4$ kNm

*Analysis using the table is easy, however it fails to provide the negative value of bending moment at middle span.*